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ABSTRACT

This document is the outcome of a project conducted in Massachusetts to apply the National Council of Teachers of Mathematics' (NCTM) "Curriculum and Evaluation Standards for School Mathematics" to adult basic education (ABE) learning environments. The first chapter describes the work of the Math Team, a committee of 22 adult basic education teachers who conducted the project. Four goals are discussed: review the NCTM standards and study their applicability to ABE; develop NCTM-based standards for four environments--ABE/basic literacy, adult secondary/General Educational Development, English as a Second Language, and workplace education; implement draft standards in field-based teacher research situations; and produce final standards and a report recording the teacher research process. Chapter 2 explores the characteristics of adult learners in each of these environments. Chapter 3 lists the 12 Massachusetts Adult Basic Education Math Standards and discusses ways to implement them with adult students. For each standard, a rationale, application, and summary are given. The standards cover the following subject areas: mathematics as problem solving; mathematics as communication; mathematics as reasoning; mathematical connections; estimation; numbers, operations, and computation; patterns, relationships, and functions; algebra; statistics and probability; geometry and spatial sense; measurement; and evaluation and assessment. (KC)

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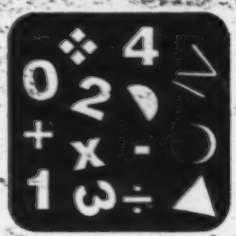
THE ABE MATH STANDARDS PROJECT



VOLUME 1:

THE MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS

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THE ABE MATH STANDARDS PROJECT

is a field-based research project for teacher development
based on the application of
The National Council of Teachers of Mathematics
CURRICULUM AND EVALUATION STANDARDS FOR SCHOOL MATHEMATICS
to Adult Basic Education Learning Environments

The main funding came as an award from
THE NATIONAL INSTITUTE FOR LITERACY GRANTS PROGRAM

to

Holyoke Community College

in collaboration with

SABES

and

The Massachusetts Department of Education

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VOLUME 1:

THE MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS

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Thanks, too, to the agencies and individuals who lent moral support, encouragement, and endorsement at key points: the SABES Regional Support Centers and advisory committee members John Comings, Myrna Manly, William Masalski, Bonnie Mullinix (also an honorary team member), and Sally Waldron.

Finally, we thank the National Council of Teachers of Mathematics whose leadership in the field of mathematics education inspired us to begin reform.

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FOREWORD

By Myrna Manly, former math editor for the GED Testing Service, has served as a member of the National Council of Teachers of Mathematics (NCTM) Adult Mathematical Literacy Task Force.

By producing this volume, the Massachusetts ABE Math Team did a good job and an important job in extending the NCTM *Standards* to the field of adult education. They have broken new ground. There are unique constraints involved with adult learning of mathematics that needed to be addressed. Most students in adult education classes have had trouble learning mathematics in the past, for some reason they missed finding the key that opens the door to understanding. By using an adult's experience as the stepping-off point, teachers can often explain abstract concepts in understandable terms. Adults may have narrow career goals in mind when they enter programs, yet they often broaden these goals when they see their success in basic instruction as the ticket to more mathematics.

Change, although inevitable, never comes easily. This is especially true in the case of reforming basic mathematics instruction for adults. For change to occur in classrooms nationwide, textbooks and tests will have to change, and teachers will have to risk change as well. The work of the Massachusetts ABE Math Team will have an impact on all three fronts.

It is well known that the majority of teachers in adult education today were not trained to teach mathematics. Some are understandably fearful of methods which allow students the freedom to explore mathematics rather than to follow prescribed rules. For teachers to risk change, they must feel comfortable with their own command of mathematics, and they must themselves have had opportunities to learn and do math that may be different from the way they were taught. The Massachusetts ABE Math Team's work shows others how adult education teachers can grow and develop as mathematics teachers. They began by studying and discussing the

NCTM *Standards* and then proceeded, as a team, to apply what they learned to the arena they knew best - adult education. Because they were a team which supported its members as they felt their way and encouraged each other through the inevitable frustrations, they made great strides with their research.

A second barrier to reform is centered around the available textbooks. Very few of these offer any new perspectives on learning mathematics. There seems to have been a closed cycle into which new approaches hardly ever entered. Users bought the books that were available and publishers kept turning out books that were similar to the ones that sold. The Massachusetts ABE Math Team also provides a ray of hope into this arena. Publishers have been alerted to their project and the participants are beginning to formulate instructional materials which correspond to their findings.

The third obstacle is that of the ubiquitous standardized testing which is used for program assessment in most facets of adult education. Presently the most-used test for this purpose in mathematics centers on isolated computational skills removed from any application. Performance on such a test cannot be a valid indicator of the progress made by students who are concentrating on the real mathematics of problem solving, pattern recognition, estimation, and exploration of relationships. Again the work of the math team will address this issue. A national conference on adult mathematical literacy will be held where practitioners such as this group from Massachusetts will be able to share their ideas with the people who make policy at the national level. Here they can discuss how stifling such tests can be to the reform -minded programs across the country and lobby for a change.

In the next few years, more progress will be made to advance these ideas of reform across the country and further steps will be take to reform the practice of adult basic mathematics instruction. As this happens, the Massachusetts team can look back at this effort and see that it was merely a beginning of a work in progress.

TABLE OF CONTENTS

Acknowledgments

Foreword

CHAPTER 1: WHO WROTE THESE STANDARDS AND WHY? 1

CHAPTER 2: WHO ARE THE LEARNERS? 13

CHAPTER 3: THE MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS. 23

STANDARD 1: MATHEMATICS AS PROBLEM SOLVING 24

STANDARD 2: MATHEMATICS AS COMMUNICATION 26

STANDARD 3: MATHEMATICS AS REASONING 29

STANDARD 4: MATHEMATICAL CONNECTIONS 32

STANDARD 5: ESTIMATION 35

STANDARD 6: NUMBERS, OPERATIONS, AND COMPUTATION 38

STANDARD 7: PATTERNS, RELATIONSHIPS, AND FUNCTIONS 42

STANDARD 8: ALGEBRA 46

STANDARD 9: STATISTICS AND PROBABILITY 49

STANDARD 10: GEOMETRY AND SPATIAL SENSE 51

STANDARD 11: MEASUREMENT 53

STANDARD 12: EVALUATION AND ASSESSMENT 56

Bibliography

CHAPTER 1:
WHO WROTE THESE STANDARDS AND WHY?

"I asked a group of my GED math students to tell me how much it would cost if I bought four shirts for $7.98 each. They were told they could figure it out any way they wanted, except they could not use paper and pencil. I watched as they used their fingers in the air or "wrote" on the desk. Most were able to multiply and get the right answer. When I asked HOW they got their answer, all agreed that they needed to multiply $7.98 by four.

"I then asked if they were in a store and had to figure out the same problem would they have done it the same way. All agreed they probably would NOT solve it the same way in "real life." Some said they would have multiplied four by seven plus four by one and then subtracted eight cents from that total. Others said they would have rounded $7.98 to $8.00, multiplied that by four and then subtracted $.08 from the product. I then asked why no one admitted to solving the problem like that in class. The response was that this was math class so they needed to do it out."

Marilyn Moses, GED teacher,
Brockton Adult Learning Center

Occurrences such as this make those of us who teach in adult basic education programs somewhat uncomfortable with the dissonance between the skills we teach and the skills adults use. Each year, over 3 million of our nation's adults return to the "educational fold" by enrolling in adult basic education programs. Because these learners are adults with the full responsibilities of adults, we as teachers are keenly aware that their time is a valuable commodity. Therefore, it is essential that what we teach and how we teach suits their needs and helps them reach their goals, whether it's obtaining a high school diploma, getting a better job, being better equipped to help their children, or increasing their literacy or everyday survival skills.

1

The goals of our national education agenda are lofty indeed as we approach the year 2000. In fact, Goal 5 of the six National Education Goals states that every adult American will be literate and possess the skills necessary to compete in a global economy. We who teach in adult learning programs are obligated to question ourselves: "Is what we teach and how we teach preparing adult students to cope better, to reason better, and to be problem-solvers? Or, are we doing "math-class" math?"

THE MASSACHUSETTS ABE MATH TEAM

You might say we're a group who are supporting each other to examine our practice and encouraging others to do the same. There are twenty-two of us. The majority teach in adult education programs in Massachusetts; the rest are involved in training and technical assistance for adult education teachers. We came together early in 1992 at a "Math Summit" hosted by Bill Arcand the SABES coordinator at Holyoke Community College, and Mary Jane Schmitt from the state Department of Education. We decided that we'd like to make some changes in our math classes, to create a more engaging and challenging atmosphere. So, we initially agreed to meet a couple of times to see if we could come up with a project. At the first meeting someone wisely suggested we write a mission statement before we jumped off into "math project frenzy" as we teachers usually do. The statement we wrote that spring summarizes what we're all about:

2

MISSION STATEMENT

The goal of the ABE Math Team is to support practitioner development and identification of materials, methodologies, strategies and programs of instruction that encourage the learning and teaching of mathematics in a manner which is interesting and appropriate to adults and which equips them with the skills needed to achieve their goals. The team will serve as advisors to projects throughout the state and, in some cases, as project developers.

BELIEFS:

- ✔ MATH IS MORE THAN COMPUTATION. IT IS A SET OF CONCEPTS, PRINCIPLES AND RELATIONSHIPS WHICH SERVES AS A POWERFUL SYMBOL SYSTEM AND TOOL FOR DESCRIBING AND ANALYZING OUR WORLD.
- ✔ THE LEARNING OF MATH FACILITATES NOT ONLY THE DEVELOPMENT OF REASONING AND COMPUTATION ABILITIES, BUT ALSO PROMOTES THE DEVELOPMENT OF SELF-ESTEEM, CONFIDENCE AND ESSENTIAL CRITICAL THINKING AND COMMUNICATION SKILLS.
- ✔ EVERYONE CAN LEARN MATH; THEREFORE, ALL ABE MATH LEARNERS SHOULD HAVE ACCESS TO THE TOTAL RANGE OF MATH DISCIPLINES AND TECHNOLOGIES.
- ✔ LEARNING IS UNDERSTANDING. THE EXPERIENCE OF DISCOVERING MATH CONCEPTS, PRINCIPLES AND RELATIONSHIPS ENERGIZES LEARNING. THE FACILITATION OF LEARNING THROUGH DISCOVERY IS, THEREFORE, THE MANDATE OF THE ABE MATHEMATICS PRACTITIONER.
- ✔ ADULT LEARNERS APPROACH THE EDUCATIONAL EXPERIENCE WITH VALUES, EXPECTATIONS AND GOALS. THEREFORE, WITHIN THE FRAMEWORK OF THE RESPECTIVE ABE PROGRAM, ALL LEARNERS SHOULD BE ACTIVE

3

PARTICIPANTS IN THE SETTING OF PERSONAL LEARNING OBJECTIVES AND DETERMINING MEASURES OF SUCCESS.

GUIDING PRINCIPLES AND CRITERIA:

1. In concert with the vision of The National Council of Teachers of Mathematics (NCTM), ABE mathematics instruction and learning should encompass not only computational tasks, but also include the essential skills of mathematical communication, reasoning, problem solving and critical thinking.
2. Materials and methods developed for ABE instruction should attempt to:
   a. Ensure that all learners perform tasks at which they can experience success.
   b. Use language as part of mathematics to explain ideas, concepts and terminology that can be understood by learners and applied by them in talking, reading, writing and listening.
   c. Provide the problem solvers with opportunity for discussion, asking questions, explaining their reasoning to others, and working cooperatively in pairs or small groups.
   d. Use practical activities and hands-on material.
   e. Teach concepts in a context relevant to adults, drawing from their backgrounds, interests and experience.
   f. Enable participants to learn about other content or topics while doing mathematics.
   g. Ensure that all instruction is free of cultural bias in any form.

This shared written belief system has guided all our work as a team. We don't think we could have accomplished what we have without it.

4

We began some small math projects that SABES and the Department of Education would support with a small pot of funds or we'd just do on our own time. Tricia Donovan began a newsletter, *the Problem Solver*, Esther Leonelli and Linda Huntington wrote a math unit "Are You a Square?" for the statewide newsletter. The team ran a "math make-it-take-it" table at the statewide adult ed conference where many teachers stopped to make tangrams, played strategy games, make manipulatives for their classes. We didn't have the means to meet very often, so we began to communicate via Mathnet, the Department of Education's toll-free e-mail network. We were very interested in the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics*, so, when the National Institute for Literacy put out a request for proposals, Tom Macdonald said, "Why don't we use this as an opportunity to see if these standards make any sense in the ABE, GED or ESL classrooms?" Thus, the main idea for the **ABE Math Standards Project** was born. The team pitched in to write and our proposal was accepted. From November, 1992, through October, 1993, this project became the team's main focus and provided funding for travel, meetings, trainings and development work.

THE GOALS OF THE ABE MATH STANDARDS PROJECT

The purpose of the project is to address the application of the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* to adult basic education learning environments. To accomplish this, we :

- ✔ reviewed the NCTM K-12 standards and studied their applicability to ABE instructional environments;
- ✔ developed an initial draft NCTM-based standards for four adult basic education instructional environments: Adult Basic Education (ABE/basic literacy), Adult Secondary Education (ASE/GED), English as a Second Language (ESL), and Workplace Education;

5

- ✔ implemented selected draft standards in actual programs in field-based teacher research situations;
- ✔ produced and disseminated two documents:

  The Massachusetts Adult Basic Education Math Standards, this volume, which is a revised version of the initial draft in light of the implementation findings; and,

  Implementing the Massachusetts Adult Basic Education Math Standards: Our Research Stories, a companion volume which records the teacher-research process, documents selected findings and provides sample teaching strategies resulting from the project.

The project reflects a focused year-long effort to improve the teaching and learning of mathematics in our own classrooms in the hope that our explorations would encourage other adult education teachers to rethink and reform math instruction and assessment to better meet the needs of the current adult population.

HOW WE ACCOMPLISHED THE PROJECT GOALS

Goal 1. Review the NCTM K-12 standards and study their applicability to ABE instructional environments.

Our first step was to learn about the *NCTM Curriculum and Evaluation Standards for School Mathematics.* Everyone on the team got a copy. We formed four small workgroups (of five) based on the population of adult learners we had the most experience with as teachers. Each group of five studied the Standards through a different lens. The ABE/Basic Literacy workgroup concentrated on adults with very basic reading and academic skills. The ESL workgroup considered how the Standards might play out for all levels of limited-English proficient adult learners. The GED workgroup thought hard about whether the Standards spoke to adults who were seeking a high-school credential, and the Workplace workgroup grappled with learners who enroll in basic education and training in the context of their workplace.

During this initial period of reading and discussion, we did some other things to bring ourselves up to speed on what was going on in math reform in schools. We retreated for a two-day intensive institute where we, as learners, spent time in cooperative groups exploring open-ended math problems. That was invaluable because it gave us a chance to experience and reflect upon learning math in the ways the *Standards* talked about. There's an adage,"teachers teach the way they were taught." How would we make changes in our classes if we never personally experienced the benefits of learning in new ways? We also made use of other NCTM resources. Everyone got a year's subscription to "Arithmetic Teacher," and we began to attend the regional conferences. We discovered a wealth of interesting instructional materials from publishers (eg., Dale Seymour, Creative Publications, and Delta) that do not normally market to adult educators and began a circulating lending library that traveled in the trunks of our cars so we could swap whenever we met.

Some teachers got the additional perk of math support from Leslie Arriola, who mentored team members on site or "on the phone" throughout the project.

Goal 2. Develop an initial draft NCTM-based standards for four adult basic education instructional environments: Adult Basic Education (ABE/ basic literacy), Adult Secondary Education (ASE/ GED), English as a Second Language (ESL), and Workplace Education.

Team members Esther Leonelli and Ruth Schwendeman served as the product coordinators and were responsible for shepherding the development of a document that recorded the team's year-long effort. They asked each workgroup to write a description of its learners and the learning environment. Then, starting with the 5-8th grade standards because they seemed to deal with the levels of basic math that most of us teach, and referencing the K-4 or 9-12 standards when appropriate, each workgroup would study each of the Standards,

7

decide whether that standard would be appropriate for their target population, then decide what adaptations should be made and why the standard was important for adult learners. Esther and Ruth synthesized the output of the four groups and developed a draft set of ABE Standards upon which we would base the next phase: making changes in our classes.

Goal 3. Implement selected draft standards in actual programs in field-based teacher research situations.

After the first draft of the ABE Math Standards was compiled, each of the team members faced the task of implementing these "lofty" ideas in our own classes. Some of us had been experimenting for a few months. Lee Thomas was creating theme-based lessons integrating math/science/social studies/reading every Tuesday; Karen DeCoster was connecting science and math in her GED class. They were finding MATHEMATICAL CONNECTIONS everywhere.

Our journey as researchers proved to be transformational in ways we had never expected. Only one of us, Martha Merson, had been involved in "action research" as an adult educator. She had worked in Philadelphia in the Adult Literacy Practitioner Inquiry Project funded by the National Center for Adult Literacy and brought an initial framework and techniques that helped guide our work.

The first step was to FIND A QUESTION that was personally interesting and that would allow us to explore the effects of changing the way we normally did business. Phrasing the question like, "What would happen if...?" got us started as explorers who were watching and listening honestly what our students were doing and saying. Having a question freed us to judge our success by how well we observed, not by how well the lesson or unit "worked." Most of our original questions evolved and narrowed as we went along, but having a question kept us nicely focused.

8

The questions we chose referred to the Standards both in content area and in philosophy. In addition, the examples demonstrate how consciously we set out to learn more about learning:

- Will learning about statistics and probability enable my students to view their world more critically and encourage them to ask more questions?
- What kinds of mathematical skills do students use when they make a quilt in math class?
- How can I facilitate the students' transition from passive learners to active agents of their mathematical learning process?

In Volume 2, the stories of these questions unfold in the teachers' own words.

Next, we made plans for what we would do to change our math classes. Some of us changed the social environment: Barbara Goodridge's previously individualized GED class began working in small, cooperative groups. Linda Huntington's basic literacy class took on the atmosphere of a quilting-bee. Others, like Deb Richard and Peg Fallon, changed the focus from sole pencil-and-paper computation to creative uses of concrete manipulatives. Sally Spencer and Ken Tamarkin would experiment with new forms of assessment. In all plans, understanding was emphasized over rote learning.

Then we made plans for how we would keep track of what happened. We collected data through a variety of ways. Several of us audio-taped our classes. Most of us kept journals, jotting notes down right after class. We took pictures, saved student work, conducted individual interviews or class discussions. Sally video-taped "think aloud-protocols."

We began to share data with one another, both at our full-team meetings and in smaller workgroups. Again, Martha and Susan Lytle from Philadelphia, offered ways to share data and to begin to look at possible themes that recurred throughout the data. While each of us explored different routes, our findings overwhelmingly confirmed our early hypothesis that by

implementing the standards, we would improve the quality of our classroom environments.

Each of us wrote a paper to record the full story of our research. We held an all-day symposium to share our findings with one another and with our advisory group.

Goal 4. Produce and disseminate two documents:

The Massachusetts Adult Basic Education Mathematics Standards, which is a revised version of the initial draft in light of the implementation findings; and,

Implementing the Massachusetts Adult Basic Education Mathematics Standards: Our Research Stories, which records the teacher-research process, documents selected findings and provides sample teaching strategies resulting from the project.

This volume, *The Massachusetts Adult Basic Education Math Standards* synthesizes our vision of the valuable components of any curriculum for adult learners in ABE, ESL, GED and workplace education classes. We start out by describing the characteristics of the adult learners we teach and then put forth eleven curriculum and one evaluation standards.

The second volume, *Implementing the Massachusetts Adult Basic Education Math Standards: Our Research Stories*, is a compilation of our teacher-research papers, each one telling the story of our journeys within our classrooms as we attempted to implement our vision.

10

OUR FUTURE AGENDA: WHAT STILL NEEDS TO BE DONE?

As far as our math team is concerned, this is just the beginning. There is much that needs to be done in order for our vision of quality math education for all adult learners to become a reality. There are few instructional materials on the market that encourage communication, reasoning, and problem solving in realistic settings. Good **materials** that help implement the standards need to be developed. Then, **staff development**

that encourages teachers to take off their expert hats and begin to learn math with other teachers and students is needed. We've begun by developing some "hands-on" workshops for our peers.

We're motivated to create **assessment** which does a better job at supporting learning. Finally, we want to continue as **researchers** in our classrooms.

We encourage all ABE/GED/ESL math teachers to join NCTM and attend their conferences. For more information and to form your own local support teams, write The National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091. Change is difficult. It takes time. But, change is easier together than alone.

We ask readers of these documents to let us know your reactions by dropping a note to: The Massachusetts ABE Team c/o Mary Jane Schmitt, Massachusetts Department of Education, 350 Main Street, Malden, MA 02148. We are also interested in meeting with other teachers interested in reform, and have developed a series of "hands-on" workshops as a result of this project.

11

CHAPTER 2:
WHO ARE THE LEARNERS?

MATHEMATICS LEARNERS AND LEARNING ENVIRONMENTS

In adult education circles, the terms "ABE" and adult basic education, are often used interchangeably with each other and in ways that may be confusing to readers outside the field. In our own work, the Massachusetts ABE Math Team first chose to use the term "ABE Math" to denote the entire range of math instruction from basic literacy level to GED or high school equivalency, and to include math instruction to non-English speaking adult learners. However, in discussions in the smaller workgroups, "ABE Math" was used to denote that level of basic skills below the GED level and not including ESL students. In order to avoid confusion, we use the term "adult basic education" when referring to the **entire** class of adult learners served by adult basic education practitioners and use the acronym "ABE" in a more limited scope. Further clarification is given below:

adult basic education: As defined by the Massachusetts Department of Education, and used in this work, this term includes the full range of basic literacy, adult secondary, and English as a second language services required by under-educated and/or limited English proficient adults, sixteen years of age or older, who are not enrolled in a K-12 school program.

ABE: This term refers to the teaching of basic literacy and basic arithmetic skills in English. Some funding sources designate two levels: ABE Beginning: Grade Levels 0-5.9; ABE Intermediate: Grade Levels 6-8.9, although grade level designations are used as a matter of expedience, not belief, and generally refer to reading scores on standardized tests. We use the term to encompass both levels of ABE instruction generally and without regard to formal testing.

ASE: This stands for "adult secondary education" and refers to Grade Levels 9 - 12. This term generally refers to that level of adult instruction which is preparation for a high school credential, such as the GED, ADP, EDP *(see below)*.

ESL: is "English as a Second Language" and refers to language instruction (oral and written communica-

tion) to non-English speaking or limited English proficient adults. The term, as we use it, includes all learners from all levels of instruction from beginning English to advanced English without regard to specific student performance levels or assessment standards.

ADP or EDP: This stands for Adult Diploma Program or External Diploma Program, an alternative high school credential program for adults developed and administered through local education authorities.

GED: This term stands for the Tests of General Education Development prepared by the GED Testing Service of the American Council on Education and administered locally under the authority of the each state department of education. The five GED tests serve as the basis for granting a high school credential to adults. The current version was introduced in 1988 and includes a mathematics test.

In keeping with the spirit of the NCTM standards, we also adopt the following definitions of "standards" and "curriculum":

standards: Standards are statements about what is valued. They are to be used to judge the quality of a curriculum or methods of evaluation.

curriculum: This term refers to an operational plan for instruction that details the content and the sequence of the mathematics taught, and includes objectives for learner achievement and activities for teachers for facilitation of learning.

THE ADULT BASIC EDUCATION MATHEMATICS LEARNER

The four ABE Math Team work groups defined themselves as instructors of the seemingly diverse adult populations of English as a Second Language learners, Adult Secondary Education/GED students, Workplace Education participants and ABE/literacy learners. However, when the editors of this draft sorted out the learner profile data gleaned from the work groups, we saw that among and between these groups certain characteristics are shared by all the learners which have a definite impact on how they learn math and what math they need to learn. These characteristics contain impli-

cations for any curriculum planning and methodology inherent in fashioning a new vision for mathematics instruction in adult basic education.

Across groups, all adult learners are goal-centered. That is, they want to be able to survive better in society: they want a second chance at a high school diploma, to get off welfare, to get a job, to get a better job, to help their kids or just improve themselves. They are from all age groups, but with most being represented in the workforce entry-and-exit age range of 18-60. Some, especially in the case of workplace education, already have high school diplomas, but feel the skills they learned (or had trouble learning) as much as a generation ago no longer serve them in the jobs they hold today.

The vastly diverse ESL population frequently has learners who hold advanced degrees in their country of origin. Others come from countries where access to basic skills learning is marginal or totally absent. Young, native born Americans who did not complete high school are taking classes because they quickly discovered the limitations of trying to succeed with too little education. Some learners in adult basic education classes often enter programs with the hope that a GED diploma will be quickly earned, even though their entry level skills, as measured on standardized tests (such as the TABE, Test of Adult Basic Education) are below the middle school level. Above all, these goals center on the world of "now," unlike the more future-oriented, child-centered world of K-12 instruction. While these personal goals are shared by most basic education students, they are not the only factors which influence the math learner's desire to perform.

Much of the math instruction that takes place in the adult basic education environment is defined by societal benchmarks. Programs are driven by outside requirements: success is determined not by such standards/goals as those detailed in the NCTM vision, but by the number of GED diplomas awarded, increases in grade level achievement, or total job placements. Public

Assistance recipients are increasingly required by federal and state governments to attend basic skills classes. In the midst of such pressures, it can be difficult to teach math in the spirit of cooperation, where various learning rates and styles are acknowledged, and where the "right" answer is not always the one at the back of the book or on the test.

Beyond such formalized external pressures lie the additional concerns and conflicts adults at any educational level face every day. Child care issues, health problems, transportation needs, housing concerns and economic instability can be daunting hurdles which frequently bar the way to the learner's achievement of educational goals. In an average adult basic education class there may be students who cannot physically or mentally attend to learning because the medication they are given impairs their performance, or because they have just come from working two full-time jobs back-to-back. Others come to class from extremely dysfunctional family situations, while some are homeless.

Adult learners who participate in ABE math, GED math, or basic skills classes attend on average from one to three hours per week. The number of weeks of instruction varies from site to site. Skill levels range from beginners with almost no math education (or sometimes any education), to college graduates from other countries. Class sizes are small in comparison to public school standards—ten to fifteen students. However, the wide range of abilities often found in a single class provides a true challenge for the adult basic education instructor. In most settings, attendance is voluntary, although attendance rules are set by the learning provider and vary from program to program. Absenteeism, related to many of the problems listed above, impacts the rate at which a student learns, the consistency of the group, and the planning and preparation efforts of teachers. Instruction is usually, but not always, individualized, or is conducted in small groups. Use of published "adult education" math workbooks is frequently the norm, but some teachers prepare their own and supplemental materials. Most of the published

materials currently available focus on computation problems. In addition, these materials assume a level of reading ability on the part of the learner that is often too high and seriously limit students' access to math learning. Some teachers make or buy math manipulatives; very few teachers have access to one or more computers. Use of calculators varies from site to site. Often math education is considered an "add-on" to basic skills instruction in reading and writing. Some programs use the reading score from a standardized test to assess readiness for math instruction, while most use a more specific math skills assessment for placement.

FOUR INSTRUCTIONAL ENVIRONMENTS

While adult basic mathematics learners have a lot in common, they also have many differences: the setting where they receive instruction, the functional abilities and life experience they bring to the class, and their personal vision of what the class should be. These students usually find themselves, or are placed, in one of four instructional environments.

1. THE ABE/LITERACY MATHEMATICS CLASSROOM

The ABE learner is usually an English-speaking individual whose reading level ranges from literacy level to pre-high school level, but whose math ability can range anywhere from pre-computation skills to secondary level math. In the last few years there has been an increase in the number of English as a Second Language individuals in the ABE math class, who may or may not have been schooled in math in their native countries, but whose reading level in English places them in the ABE level class. Among native English-speaking students, most of these learners have not completed high school; some may have learning disabilities. For many, there is a high dependence on auditory or tactile learning because of low-level reading abilities. Some are concrete learners who have underdeveloped abilities in abstract reasoning. ABE learners may be self-confident or have low self-esteem. Their educational goals (to get the GED in a relatively short time, for instance), may or may not be in line with their incoming basic skills level.

At the same time, learners in this group (like nearly all adult basic education learners) bring a wealth of motivation and courage to the task of learning math. Such determination makes teaching the adult basic education student a continuing pleasure.

Settings for ABE programs vary: church basements, libraries, community organizations, public schools, college classrooms, etc. Sometimes math instruction is a one-day-per-week affair incorporated into reading and writing instruction, sometimes it may be a separate class by itself and meets several times a week.

2. THE ENGLISH AS A SECOND LANGUAGE MATHEMATICS CLASSROOM

The ESL math learner population falls into three groups: a) non-native English speakers enrolled in ABE/pre-GED/GED classes who can communicate effectively in oral English and may be integrated into ABE classes with native speakers; b) students in formal ESL classes where they may be learning everyday survival math but are not enrolled in a formal math class; c) students enrolled in native language literacy classes where math, if taught, is done so in the native language. This group also includes learners who are preparing for the GED in Spanish or French.

Instruction for all these groups is framed by specific expectations of both the learning provider and the learners themselves. While everyone would agree that math learning is the "third basic" for any non-native ABE participant, it is often the last essential to be addressed, given the learner's language limitations. Unless classes are bilingual, direct instruction in mathematics for the ESL learner is usually postponed until the oral/aural language skills are developed. Unfortunately, as ESL learners begin to gain access to the institutions and social settings of the English speaker, real life math-related situations arise which the individual is unable to verbally address—he/she does not know or cannot use the "language of math."

Often culture-based math learning experiences determine the way ESL learners expect to approach

math as adults. A pre-conceived image of "teacher-as-expert," the gate-keeper of knowledge, may inhibit the ESL math learner's willingness to use inherent intuitive insights or to apply life experiences and common sense to problem solving situations. Leslie Arriola, an adult basic education teacher, notes:

"The dependency on the teacher is... painfully typical.... Adults who are normally exploratory and inventive in how they solve everyday problems seem to leave these reasoning tools at the door of the mathematics classroom. Their view of math does not include invention and exploration. Math is cut and dried, something that has rules and methods that have to be memorized. And their view of math certainly doesn't include socializing. Talking to other students about how to do a problem means you're not smart enough to solve it on your own. If you're stumped, ask the teacher. If you want to know if you're right, ask the teacher."

Rote learning, text-specific math experiences are often what the ESL math learner seems to need or want, based on his/her own education experiences, and any instruction such as group work or examination of multiple approaches to solving the same problem might be suspect. (It should be noted that this phenomenon is not isolated to only the ESL math learner; many English speaking natives have experienced the same type of "teacher-centered" math instruction.)

3. THE ADULT SECONDARY EDUCATION/GED MATHEMATICS CLASSROOM

ASE/GED learners include individuals in GED preparation classes or adult education programs called "Adult Diploma Programs" or "External Diploma Programs," which grant an alternative competency-based, high school credential. Passing the GED or the ADP/EDP math competency test is the primary goal of this math student. The objectives of developing a problem solving attitude, being able to communicate in mathematical terms, reasoning mathematically or seeing mathematical connections in everyday life are, for this

learner, often secondary at best. Ironically, employers are now requiring the very problem solving strategies adult diploma candidates don't view as important. ADP test developers Kenneth Tamarkin and Susan Barnard comment on their experience:

"The premise of the ADP is to earn a high school diploma by demonstrating proficiencies in life-skill based competencies that have components of reading, writing and math interwoven throughout the curriculum... Over the years, we have identified the weakest skill area to be critical thinking. We found that we can teach to the ADP math entrance test and students can quickly pass. But given a similar calculation, or problem out of context or with a change in the format, students are not transferring their skills."

At any rate, most ASE/GED learners enter the adult diploma class resolving to buckle down, pay attention and succeed this time; to undo unsuccessful past educational experiences; or sometimes to fill in the learning gap between schooling received many years ago and the survival demands of today's world. This newfound resolve can be tenuous and may be tested by life situations which often seem insurmountable: child care problems, racial discrimination, poverty, loss of employment, health problems, imprisonment, and others. The learner's resolve to "play-by-the-rules," and pass the test may be fragile. In the face of these obstacles, it is a continuing challenge for all ASE/GED instructors to offer learners substantive math skills that will serve for a lifetime.

4. THE WORKPLACE EDUCATION MATHEMATICS CLASSROOM

In some ways, the math learner in a workplace environment has distinct advantages in contrast to many learners in community-based settings. First, of course, he/she has a job, which can have enormous implications for the learner's self-esteem and belief in his/her ability to learn. Second, the company (frequently in tandem with the employee union) provides and supports this on-site education — the learner does not need to

travel to take advantage of basic skills training. There is often pay linked to attendance in such classes. And finally, workplace students have the chance to immediately apply what they have learned by practicing newly formed skills back on the job. With all these advantages, however, the workplace environment also carries with it implicit messages to learners which can be worrisome or even frightening. Students/employees can see the writing on the wall: the skills they have currently may not be enough to help them keep their job in the near or distant future. Many workers have held the same or similar jobs for years or even decades; they see their company moving into the age of high technology and are fearful of being left behind. Often the skills they received via middle school, high school, or in some cases post-secondary education do not help them feel competent when it comes to doing statistical process control charts, or checking tolerances to three decimal places. They are embarrassed when faced with these situations; they are equally as dismayed when supervisors suggest they upgrade their skills.

Workplace math learners come from all functional education levels: some can't read or perform the four basic math operations, others don't speak or read English well, some never got the high school diploma and want to get a GED. But all need to be able to do the math related to their jobs with more accuracy and facility. Learning providers respond to this by examining closely the daily math tasks required on-the-job. Math materials are either purchased or created which directly address these requirements. Because workplace priorities frequently come before classroom priorities, attendance can sometimes be a problem. This, and the broad range of abilities stated earlier, prompts many instructors to work individually or in small groups with workplace learners. Class schedules are formed around the work schedule, and math programs can be victim to the same vicissitudes a company can face: downturns in the economy and loss of company profits can signal the end for job-related math classes.

CHAPTER 3:
THE MASSACHUSETTS ADULT BASIC EDUCATION MATH STANDARDS

The Massachusetts ABE Math Team, after a year of study, reflection and practice, proposes twelve standards to be used to inform and guide mathematics teaching in adult basic education. These twelve standards are adapted from the 1989 National Council of Teachers of Mathematics document, *Curriculum and Evaluation Standards for School Mathematics.*

The NCTM publication includes curriculum standards for grades K-4 (14 standards), grades 5-8 (13 standards) and grades 9-12 (14 standards). It also provides a set of standards (14) for the area of mathematics evaluation for grades K-12. The Massachusetts ABE Math Team studied and discussed each set of standards and developed the following set of twelve adapted standards, a hybrid distillation of the broad range of curricular goals found in the NCTM K-12 standards document.

For each standard, there is an introductory narrative which provides a *rationale* for the inclusion of each topic. A discussion follows which outlines in general or specific terms the *application* of the standard to the four adult basic education mathematics learning environments. Interspersed within both sections are quotes, vignettes or anecdotes which illustrate the concept or methodology of the standard. These are taken from the teacher research phase of this project: twenty teachers doing field-based research to implement the newly revised standards. *(For complete documentation of this research, see Volume Two of this publication.)* A final *summary* highlights the essence of the standard.

The first four standards and the final evaluation standard are the core of the Massachusetts ABE Math Standards; they form the basis for all recommended methodologies which follow. The remaining standards deal with individual content areas. Our hope is that the final publication of this document will serve as a point of departure not only for other teachers who are rethinking and reshaping their mathematics teaching, but for publishers, test-developers, and adult basic education funding agencies across the nation as well.

Much of the text of the Standards is very close in wording to the original NCTM text because of the alignment we share with NCTM goals. However, where appropriate new phraseology has been added to accurately reflect the differences between teaching K-12 students and the adult basic education population. We are grateful to the NCTM for providing an inspiring and motivating model.

STANDARD 1:

MATH AS PROBLEM SOLVING

RATIONALE

Problem solving in today's world is not a simple task. We are faced with a fast-paced, complex, multi-dimensional universe where we must continually develop new skills and abilities. The need for new strategies for problem solving is real; and never so real as in the mathematics problem solving adults must be able to accomplish every day. Critical thinking skills must always accompany any form of computation. Real facility with calculators and other technologies is becoming a commonplace expectation. Employers demand that workers be able to quickly apply basic skills knowledge to a variety of ever-changing job situations. Adult basic education learners face these requirements daily. They know through experience that yesterday's mathematical proficiencies do not meet the requirements of today. Problem solving methodologies in the mathematics curriculum must help learners meet the real needs of the real world.

APPLICATION

Adult basic education mathematics teaching must involve authentic tasks centered in authentic problems using multiple problem solving strategies. Isolated, non-contextual computation drills are not enough. Teachers must engage learners in true real-world problems. ABE instructor Cathy Coleman posed a question to her class: how could she determine whether or not her car used more gas when she drove using the air conditioner? Students came up with a variety of solutions:

Student One: Check the gas tank before you leave, then see how much you have left after.

Student Two: I heard you save gas if you unplug the air conditioner. Drive for awhile with [it on]. Then unplug it for another few weeks. See if you get better mileage.

Student Three: Fill up the tank and see how long it takes to use the gas [without the] air conditioning. Find out how much gas you use with the air conditioner. In my opinion you will use more with the air conditioner. How much more, I don't know. You will have to do the test.

Good problem solving techniques allow the learner to develop essential critical thinking skills, confidence, and important communication abilities.

Mathematics instruction must also provide the adult basic education learner with multiple opportunities for success in problem solving. A lifetime of negative experiences or memories of the educational process has produced in many students a major lack of self-esteem and self-confidence. This lack prevents the individual from attempting important risk-taking tasks involved in thinking and communicating about mathematics.

Instructional strategies must include methodologies which are framed by the broad empirical and cultural knowledge base of each learner. Adult students come to the basic mathematics classroom with a wealth of non-standard problem solving strategies which must be recognized and given credibility. And at all turns, the goal-centered nature of the adult mathematics learner must be acknowledged and accommodated; when this student cannot not see his problem solving experience as moving him closer to his objectives, the risk of failure is high.

SUMMARY

In the adult basic education classroom, curriculum design must include approaches which allow the learner to:

- ✔ explore and employ multiple strategies for solving problems;
- ✔ determine, collect, and analyze appropriate data with respect to the original problem or in new problem solving situations;
- ✔ have access to and the ability to use appropriate problem solving tools including the use of calculator, computers, and measurement instruments;
- ✔ generalize problem solving strategies to a wide range of adult-oriented, real-world situations.

STANDARD 2:
MATHEMATICS AS COMMUNICATION

RATIONALE

Instructor Tom Macdonald talks about communication in his ESL mathematics classroom:

"I tried to have my students work in small groups so they could begin to break their habit of approaching the solving of math problems in solitude, so that they could develop together a process of reasoning and elimination, and so they could share in the excitement and satisfaction of getting to the solutions. They spoke to each other sometimes in Kreyol, and sometimes in English. One student in particular had a very difficult time even beginning to unravel the problem. He did not do well working with anyone else. He did, however, very carefully draw four houses with a ruler. I was pleased with his diagram, but concerned that he was stymied in getting past his first step in his problem solving process.

"When [another group] finished [the same problem], I asked the him to join them so they could explain their solution. He listened to them with the same sort of blankly puzzled expression he had shown once his houses diagram was drawn... Finally he latched onto a piece of the explanation. As he began to understand...the clues fell into place. A smile slowly lit up his face. I quite literally heard math wheels begin to turn inside him."

Much of the mathematics adults encounter everyday demands interaction between two or more people: in the workplace, at the grocery store, at the bank, and certainly in the homes of every adult dealing with the day-to-day demands of keeping a household financially together. "Doing math" may begin with a solitary pencil-on-paper computation activity such as completion of a time sheet or filling out a tax form, but in the real world seldom does the experience stop there. In the adult basic education classroom, learners do need time and silence to practice and sharpen their performance of necessary algorithms (computation or other math procedures), but non-stop worksheet computation allows many learners to become obsessed with getting the (only) right answer.

This limited exchange between the student and the answers in the back of the book makes math learning an isolated experience that is clearly at odds with the real ways math is used. In order to organize their daily lives and provide for their children, adults must be able to comprehend, discuss, and act on our many intersections with math — from taking a temperature to analyzing figures on life insurance charts.

APPLICATION

Given such immediate real world demands for math usage, it is essential that the mathematics curriculum of the adult basic education classroom include strategies which promote skill development in shared problem solving and communication of mathematical ideas.

Barbara Goodridge's ABE class struggled with learning how to share what they knew:

"I [Barbara] heard mistakes as two students discussed a problem. Another member of their group, a quiet Cambodian man, tried to tell them [they were in error], but they didn't acknowledge him. He tried to interject twice. Finally, as they finished with the wrong answer, one of them asked if they all agreed. Quietly, but with confidence, the man said, 'No,' and showed them their error."

Adult education math learners must not only learn to discuss their problem solving ideas, but see that math can be communicated in many other forms as well. Extensive use and production of written and graphic mathematical representations is essential. ABE/ESL teacher Lee Thomas shares her experience:

"The learners' inquiries into the formation of graphs lead to more work...they were instructed to bring in graphs of interest to them....There were bar graphs on everything from consumption of candy bars per person per year, to circle and bar graphs on the concentration of wealth in the United States... and most important, how the less educated were suffering more. This last one had a great impact upon my learners, for they noted the impor-

tance of a high school diploma and its connection to a better income."

Talking through or writing about the real ways math portrays our world provides the learner with essential skills for critically thinking about issues important to each of us.

In order for practitioners to facilitate implementation of this or any of the other standards outlined in this document, we suggest that publishers re-examine materials written for the adult basic education classroom. Beyond the concern of too many pages of rote computation is the question of access. Often adult education publications are written at a reading level much too difficult for a good portion of the population they serve. Use of idiomatic language slows down or stops non-English speakers. And inexplicit definition of mathematical terms which also have common English meanings (i.e. the geometric idea of a plane versus the vernacular use of the word "plane") can also be a communications barrier for the adult learner.

SUMMARY

In the adult basic education classroom, curriculum design must include approaches to teaching mathematics as communications which allow the learner to:

- ✔ develop appropriate reading, writing, listening and speaking skills necessary for communicating mathematically in a variety of settings;
- ✔ discuss with others, reflect and clarify their own thinking about mathematical outcomes, and make convincing arguments and decisions based on these experiences.
- ✔ define everyday, work-related or test-related mathematical situations using concrete, pictorial, graphical or algebraic methods;
- ✔ appreciate the value of mathematical language and notation in relation to mathematical ideas.

STANDARD 3:
MATHEMATICS AS REASONING

RATIONALE

Reasoning is key for determining problem solving strategies, evaluating alternatives, and analyzing results whether on the job or on the GED. Adults must be able to develop and present reasonable convincing arguments in the context of many real world situations: as consumers, as employees, as students, or as family members.

Most adult basic education learners are developmentally able to reason abstractly, but many have had little opportunity to practice formalized reasoning while doing math. Few adult learners have experienced classroom activities beyond a textbook with paper-and-pencil exercises.

Adult basic education mathematics instruction must help the adult learner know that he/she has the power to do mathematics, and has control over the success or failure of that effort. This power cannot exist unless the learner uses reasoning to justify his/her own thinking. Students need to know that mathematics makes sense, is logical and even enjoyable. A class that values reasoning also values communicating and problem solving.

Instructor Cathy Coleman's describes the reasoning activities and outcomes of a recent GED class:

"All four [of my] students had difficulty with the "egg" problem:

There are fewer than six dozen eggs in a basket.

If I count them two at a time, there is one left over.

If I count them three at a time, none are left over.

If I count them 4, 5, or 6 at a time, there are always three left over. How many eggs are there?

"Different people used different methods to test their solutions. One student employed the strategy of using tally marks to count by two or three, etc. Others divided by two or three using paper and pencil. And another counted by five aloud in Spanish."

"No one seemed so overwhelmed and discouraged as to not try the problem. All four students were able to choose numbers based on the facts within the problem and to make new choices based on the results of their own mathematical experiences/experiments with them. I was surprised to see how much they all seemed to actually enjoy the challenge of the situation."

"Out of four students, only one has solved the problem to date [Spring, 1993]. However, three tell me they are still trying and we have been out of school for two weeks. Some took it home too and have practiced a little 'family math.' The [learner] who solved the egg problem was delighted. She told me she figured it out after trying many different numbers and after using "that thing you gave me last week." The 'thing' she was referring to was from a book called **Those Amazing Tables.** *The goal is for students to see the patterns inherent in the times tables. It took her several hours of hard work, but [she solved the problem and] we were both thrilled."*

"Certain students in the class seemed to come alive this year. One young woman, who a year ago didn't say much at all, seemed to really become an active participant. She was able to communicate her thinking to other members of the group and help them understand problems in a different way. Another told me of her experiences with comparing prices at the grocery store. Both seem to be exercising more strongly their ability to question. The [student who solved the egg problem commented]: 'It is good to try new things and find out you can do them.'"

Genuine respect and support of each other's ideas is essential for learners to be able to explain and justify their thinking and to be able to understand that how the problem is solved is as important as its answer.

APPLICATION

In all adult basic education math settings, the development of critical thinking skills is crucial. Statements should be open to question, reaction and elaboration from others. At the same time, the role of insight or intuition should also be promoted. An atmosphere must be established where learners are afforded many opportunities to explore, to apply reasoning skills, to ask questions and to examine and validate their own thinking. They should feel free and confident to explore their own questions. It is also essential (particularly for ABE learners) that concrete materials be made available to assist learners in supporting their reasoning, whether inductive, deductive, spatial, or visual. Such reasoning is at the heart of problem solving. And finally, learners need to be encouraged to go beyond endless practice in computation in learning mathematics. Students need to see that those aren't the only important skills that will help them in their daily lives, at work, or on an adult diploma exam.

SUMMARY

In the adult basic education classroom, curriculum design must include approaches which emphasize mathematical reasoning so that the learner can:

- ✔ draw logical conclusions from mathematical situations using concrete models and verbal skills to explain their thinking;
- ✔ understand and apply deductive and inductive reasoning, proportional reasoning, with special attention to spatial and visual reasoning with proportions and graphs;
- ✔ pose their own mathematical questions and evaluate their own arguments;
- ✔ validate their own thinking and intuition, feel confident as math problem solvers, and see that mathematics makes sense.

31

STANDARD 4:
MATHEMATICAL CONNECTIONS

RATIONALE

For all adults, mathematics learning should be connected to real life situations. When mathematical concepts are linked to everyday life skills and experiences, learners become aware of the usefulness of the discipline beyond the classroom, of the importance of mathematics in all career choices, and of a useful tool applicable to the real world. Tricia Donovan's GED class made this effort a regular ritual:

"Our Monday morning classes always began with a roundtable review of everyone's weekend and a news report. Often we strayed off for a discussion of geography or politics or money or local affairs, depending on the headlines... This time for connecting always made it easier for the class to refocus their energies on the [upcoming math] class."

"On Monday as we went around the table hearing everyone's stories, we had one of those digressing moments that became a math lesson. One learner reported that she and her family had made maple syrup that weekend and had boiled down enough sap (30 gallons) to make almost a full gallon of syrup."

"'What do you mean [by] 'almost' a gallon?' I asked."

"The mere mention of a 'mathy' word like 'gallon' always provokes a question from me, and students who have been in the class for awhile will say to newcomers, "Oh, oh, you've done it now. She's going to stretch our brains."

APPLICATION

This standard speaks both to the ways math is taught to adults and to the ways the learner should relate math and mathematical thinking to real world and work-related settings. Adult education should put less emphasis on the teaching of isolated mathematics skills and increase emphasis on teaching a meaningful program that is more holistic and related to learners' daily lives. The adult basic education teacher must be familiar with the students' educational, cultural and language

background in order to connect with students' thinking, and be flexible enough to build on the different ways adults learned math as children. Making mathematical connections to a variety of topics facilitates a spiral approach to teaching which accommodates adults with a broad range of academic and literacy skills.

ABE mathematics instruction, as well as math for ESL students, should integrate teaching language skills (listening, reading, writing, and speaking) with teaching math skills. Especially at the ABE level, connections should be explored using concrete materials and appropriate technology, accompanied by discussion to develop language skills. Making such connections thus aids learners as they move from concrete thinking and problem solving to abstract concepts. It also broadens their perspective and promotes the vision of mathematics as an integrated whole. A GED student in Debra Richard's class chronicled her experience with learning about measurement through the use of manipulatives:

"[On] my first day in Debra's class, we tried to find how many ounces were in a cup. [After the class] I went home and told my husband how stupid I [felt] I was and I cried all night. [Later,] he set up bottles and measuring cups and we poured water... until I understood. I never liked math before, but I know that I can do it now."

For the workplace education student, instruction must assist the learner in seeing the connection between problem solving strategies used in the classroom and math-related tasks on the job. Teacher Judi Sulzbach describes a scene from one of her workplace math classes:

"I wanted [them] to see the relationship (connection) between the metric system, [decimals] and English measurements, so if need be, [they] could do the conversions with paper and pencil. Students were working with grinding wheel tolerances (measurement ranges] that are allowed without affecting wheel dimensions, showing them as decimals, fractions and in metrics. One of the learners said that

he never realized that 3.25 and 3 1/4 were the same. He had looked at a chart of hole sizes for years and always thought that 3.265 and 3 1/4 were the same, because that was [the tolerance] figure his chart showed."

The ability to see mathematical connections is essential in order for employees to become more flexible, efficient, and productive.

SUMMARY

In the adult basic education classroom, curriculum design must include approaches to making mathematical connections which allow the learner to:

- ✔ View mathematics as an integrated whole that is connected to past learning, the real world, adult life skills, and work-related settings;
- ✔ explore problems using appropriate technology and describe results using a variety of mathematical models or representations including graphs, concrete, verbal, and algebraic models or representations;
- ✔ apply mathematical thinking and modeling to solve problems that arise in other disciplines, and in the real world, including work-related settings.

STANDARD 5:

ESTIMATION

RATIONALE

Estimation is probably the most used and useful skill for adults and continually plays an important role in the adult learner's life. Adults use informal measurements in life skill activities such as cooking, shopping, buying clothes or estimating the time required for daily tasks. Estimation is a valuable skill for checking the reasonableness of computation or accuracy in problem solving, and is an aid in timed-test situations such as the GED. It builds on adult experience and knowledge and also helps develop number sense, operation sense, and computation skills. Good estimators use a variety of strategies and techniques for computational estimation which can be explored and shared by learners with each other. This anecdote comes from Lee Thomas's ABE class:

"[One of my students], a young high school drop-out, was trying desperately to obtain a high school diploma in order to get a job and support his daughter. He lacked self-esteem and self-confidence. The students weren't sure of him due to his [unconventional] looks. [But] during our 'gram versus ounce' lesson, a new respect for his thinking came about.

"Learners were asking each other, "What is the difference between a gram and an ounce? Which one is heavier?" They examined various plastic gram weights and then decided to weigh and compare a list of objects in the room: buttons, a feather, a pencil, a ruler. The group huddled around a table set up with a balance, gram weights, and the various objects chosen to be weighed. I asked them to estimate the weight of each object by holding them, then find the actual weight. Students took turns and others recorded the results."

"The estimates for the weight of a plastic twelve-inch ruler were very much off. The average estimate was five grams, when, in reality, the actual weight was 15 grams. The class couldn't understand how they had underestimated its weight. "It doesn't feel that heavy!" and "I can't believe I was

that far off!" They struggled for an explanation while continuing to experiment. I was excited that they couldn't let go of this problem! In the background, I could hear our young father explaining the discrepancy as he spoke to another student:

"The one-gram mass is about the same size and thickness as an inch on our plastic ruler. If you put 15 of them (the one gram mass) all lined up that would look like a ruler (side by side), so they'd be the same ...'cause that's plastic and this is plastic. You can see it, it would take at least twelve of them.'"

Ms. Thomas continues:

"I had him repeat this to the entire group. Here was deductive reasoning and logical thinking being imparted... by someone everyone had been unsure of since day one! The students now saw him in a different light and not only continued to ask him more questions, but also valued his opinions as we [worked through each] lesson. [This young man]... progressed rapidly....In fact, he was the first student in the class to begin...geometry!"

APPLICATION

In ABE math and ASE/GED math, the teaching of estimation to adults takes on a meaning beyond "rounding off" numbers as it is currently presented in many adult basic education texts. The math curriculum should include ways for the adult learner to develop an estimation mind-set when approaching problem solving in real-life contexts and in test-taking situations such as the GED. Instructors should be especially aware of the educational backgrounds of ESL students when teaching estimation (or other number skills), since many learners come from cultures where decimals rather than fractions are used for computing. In the workplace education setting, it is important that the learner have a solid understanding of estimation in order to check the reasonableness of results, since on the job those results will be used in a real situation.

SUMMARY

In the adult basic education classroom, curriculum design must include approaches to teaching estimation which allow the learner to:

- ✔ explore and develop the concepts underlying a variety of estimation techniques and strategies for whole numbers, fractions, decimals, and percents;
- ✔ recognize when an estimate is appropriate and useful in real-life situations and the role estimation plays in adult life;
- ✔ apply estimation techniques in working with quantities, measurement, computation, problem solving, and in workplace and test situations;
- ✔ use estimation to check the reasonableness of results.

37

STANDARD 6:
NUMBERS, OPERATIONS, AND COMPUTATION

RATIONALE

To be efficient workers or consumers in today's world, adults must have a strongly developed conceptual understanding of arithmetic operations as well as the procedural knowledge of computation and number facts. They must be able to perceive the idea of place value and be able to read, write and represent numbers and numerical relationships in a wide variety of ways. Simple paper-and-pencil computation skills are not enough. Adults must be able to make decisions regarding the best method of computation (mental math, paper-and-pencil, calculator/computer) to use for a particular situation. Knowledge of numbers, operations and computation must include both a well-developed number sense and the ability to use basic mathematics-related technologies.

APPLICATION

Adults in all the adult basic education learning environments bring a range of abilities, skill levels, and gaps in their learning and dealings with numbers as a result of their varied educational and cultural backgrounds. Some ABE-level adults demonstrate a limited understanding of place value and how big or small numbers are. ESL students from countries using metric measurement may demonstrate stronger skills in decimal concepts and computation while learners from the North American tradition may demonstrate greater facility with fractions. Still, to many other adults, fractions present a constant puzzle and irrelevancy to their daily life. While almost all adult learners have an understanding of the U.S. money system and a working knowledge of the underlying decimal notation, this knowledge may not translate to a deeper understanding of decimal skills and operations which are essential workplace skills.

The adult basic education mathematics curriculum, then, should include a continuum of activities and opportunities for learners to develop a strong sense of what numbers are (whole, fractions, decimals, percents, and integers), of how big or how small specific numbers are and of how numbers are related to each other.

A learner in teacher Karen DeCoster's class wrote:

"We learned today how to light a light bulb by making a complete circuit... you need direct current (DC) to make it happen. We also learned some math. If one battery has 1.5 volts and you tape 25 of them together...to see if the bulb burns brighter (it does), you can figure it out in your head that you are using 37.5 volts of power to do it. All you do is multiply 25 times one to equal 25, and then cut that figure in half since .5 means half. 25 divided by two equals 12.5. Add it all together and you get 37.5."

All adult basic education mathematics students can develop a sense of the magnitude and relative position of numbers through the use of physical materials and "real life" referents. By employing such "realia," connections between fractions, decimals, and percents can be explored, analyzed, and understood. Students in the end should be able to understand the numbers they use in their everyday lives and in the world around them.

For the ESL student, it is important to develop English language and vocabulary skills necessary to express numbers and number concepts. In particular, for students trained in the European method of notation it means learning the difference between the American use of comma and decimal point and the European use, i.e. 0.5 instead of 0,5 and 5,000 instead of 5.000.

Workplace learners must be taught how to determine appropriate vehicles for describing numerical phenomena. They should be given experience with plotting and interpreting numerical relationships in one and two dimensional graphs.

ABE math teaching should stress the development of conceptual understanding for arithmetic operations as well as the procedural knowledge of computation and number facts. This means providing the learner with opportunities to explore, explain, and develop proficiency with a variety of models for each of the four basic operations.

Computation skills should be practiced in the context of problem solving and not as a set of isolated skills. Adults should be encouraged to develop and share their own tricks and ways of computing percentages; for example, sharing short-cuts to determining the tip on a meal tab or finding a discount.

The GED/ADP/EDP curriculum should offer adult students the opportunity to develop and apply number theory concepts of primes, factors, multiples, unity, infinity, negation, order relations, and order of operations and to make the connections between number theory and algebraic symbolism. These students can also profit from cultivating an "estimation mind-set" when looking at GED questions. Estimation should be taught both as an aid in developing the learner's computational skills and as an aid to solving problems and checking one's solution. Proportional reasoning should be included as an important tool in problem solving. Testing formats for this population, whether used for assessment or certificate granting purposes, should place less emphasis on the exact forms of answers as opposed to the reasoning and reasonableness of the result.

SUMMARY

In the adult basic education classroom, curriculum design must include concrete and developmental approaches to teaching numbers, number relationships, operations, and computation which allow the learner to:

- ✔ understand, represent, and use numbers in a variety of equivalent forms and in order relations (integers, fraction, decimal, percent, exponential, and scientific notation) in real-world, work-related, and mathematical problem situations;
- ✔ compute with whole numbers, fractions, decimals, and integers, using appropriate algorithms and a variety of techniques including mental math, paper-and-pencil, calculator, and computer methods;
- ✔ analyze and explain procedures for computation and understand how arithmetic operations are related to one another, particularly the reversibility of operations;
- ✔ Use estimation to develop number sense, operation sense, and to check the reasonableness of results.
- ✔ Analyze and explain methods for solving proportions;
- ✔ select and use in problem solving situations an appropriate method from among mental arithmetic, paper-and-pencil, calculator, and computer methods:
- ✔ use computation, estimation, and proportions to solve problems.

STANDARD 7:

PATTERNS, RELATIONSHIPS, AND FUNCTIONS

RATIONALE

Patterns abound in the rhythm of adult lives: daily schedules, the school calendar, the time clock at work, the arrangement of streets and buildings. They also form the basis for most of the mathematics adults use — from multiplication tables to common formulas (distance, cost, interest relationships) to statistics and more abstract algebraic functions. Working with patterns helps adult basic education learners develop the ability to classify and organize the mass of information they encounter everyday.

Learning to recognize and analyze patterns and number relationships connects mathematics to the world. In disciplines such as literature, political science, genetics, astronomy and chemistry the skills of pattern recognition and functional description are as essential as in mathematics.

This kind of thinking skill serves as a foundation for the development of abstract ideas not only in mathematics, but across a wide range of academic topics.

APPLICATION

The study of patterns, relationships, and functions represents an ideal and essential vehicle for the presentation of an integrated mathematics curriculum for adult learners. Lessons should be extended to include real life data and life skill information, such as material from surveys or cost tables. Activities for organizing results with graphs or other representations should be used. This helps the learner identify properties and test and predict outcomes. Student interaction and verbalization (especially with ESL learners) should be facilitated through small group work and large group discussion. Reliance on intuitive reasoning and discovery should be encouraged and students should be asked to make generalizations regarding the relationships between numbers and in their patterns.

In the ABE classroom, the study of patterns should start with the concrete. At the most basic level, students can begin with counting patterns. Work can include hands-on manipulation of real objects like pattern blocks, Cuisenaire rods, rulers, etc. Instructor Peg

Fallon talks about her use of manipulatives with a beginning ABE student:

"Initially I worked individually with [this student] because his math skills were so limited. He could count by using his fingers or a tally, and had committed no number facts to memory. By using a [number facts] chart, he progressed slowly through addition, substraction, and multiplication. But division had been overwhelming for him...he seemed not to comprehend the concept of dividing into groups."

"It occurred to me that manipulatives might enable him to understand the [division] process. I began by using an egg carton containing twelve small green pattern blocks ("unbreakable eggs")." [The following is an abbreviated script of Mrs. Fallon's lesson.]

Teacher: How many eggs are there?

Learner: I see twelve.

Teacher: Yes. That's how they're packaged in the grocery store. Do you know another name for twelve eggs?

Learner: You mean a dozen.

Teacher: Sure. Twelve eggs is called a dozen... Now look at the eggs in the carton. Can you tell me how they're arranged?

Learner: You mean like rows?

Teacher: Very good. We could also say they're in groups. How many rows do you see?

Learner: (pointing) Two.

Teacher: How many eggs in each of these rows?

Learner: (counting) Six.

Teacher: Good. Do you see another arrangement of rows?

Learner: Do you mean up and down? There are only two eggs each time.

Teacher: Good. How many up and down rows are there?

Learner: Six.

Teacher: What can you tell me about two rows of six eggs across and six rows of two eggs up and down?

Learner: The number is the same.

Teacher: What number?

Learner: If you add these two [rows of six] and add all these [rows of two] you get the same number: twelve eggs.

Teacher: Do you think there's another way of getting the answer?

Learner: Do you mean multiply? Like six time two or two times six? I don't know multiplying very much.

Mrs. Fallon continues:

"In subsequent sessions we extended the activity using blocks, toothpicks, or pencils, increasing the total number gradually by twelves until we reached a total of 120. He willingly counted out and grouped elements, and despite being unable to retain number facts in his head, he was able, by means of visualizing and manipulating [concrete items], to comprehend grouping."

In all adult basic education classrooms, use of the calculator or computer should be an additional "hands-on" means for extending the analysis of patterns. Adult diploma or workplace education students can experiment with using the constant key to find the sales tax (constant) of various retail items, to convert measurements (using a constant conversion factor for changing feet to inches or metric to U.S. system), or to show exponential growth (calculating the powers of two, ten, or other base numbers). Using spreadsheet software also lends itself to demonstrating patterns and formulas. Learners can develop interest tables or sales tax tables in real-world applications of both mathematics and computer technology.

SUMMARY

In the adult basic education classroom, curriculum design must include approaches to teaching about patterns, relationships, and functions which allow the learner to:

- ✔ explore, recognize, analyze, and extend patterns in mathematical and real-world situations;
- ✔ articulate and represent number and data relationships using words, tables, graphs, and rules.
- ✔ discover and use patterns and functions to represent and solve problems.

STANDARD 8:
ALGEBRA

RATIONALE

Math concepts formerly taught only in basic algebra courses are increasingly part of the culture and vocabulary of modern life. For example, the concept of positive and negative numbers is found in widely-used medical terminology such as "HIV-positive," or "RH-negative." The concept of "exponential growth" appears in descriptions of medical or business phenomena such as the increase in the number of AIDS cases in the last ten years or the expected growth of the U.S. debt.

Life experience has afforded adult basic education learners with a broad base of real-world ties which can be readily linked to the concepts of equation, function, variable, and graph. From baby formulas to chemical formulas, algebra offers a succinct way to define real-world situations that can aid adults in the home and in the workplace. ABE teacher Esther Leonelli offers two examples:

1. A person following a popular diet plan uses color-coded cards to represent the allowable daily food exchanges. These can be connected to the "formula" for a balanced daily caloric intake:

$$3F + 2V + 2M + 3B + 3f + 8P = \text{Daily Total}$$

where F stands for fruit, V for vegetable, M for milk, B for bread, F for Fat and P for protein units.

2. A nursing home aide who plans on entering nursing training after earning his GED can learn to apply the algebraic proportionality rules used in figuring dosages and medications.

The opportunity to study algebra should be available to any adult basic education learner who may have missed it due to past educational experiences. Students whose academic background did not include algebra, or for whom the study was unproductive when it occurred, marvel at themselves when they, as adults, finally "get" what algebra is all about. As a matter of equity, algebra instruction should be made accessible to all adult learners, whether they were previously denied access by public school "tracking," or currently by adult education

models of instruction which preclude the study of algebra as a "life skill."

APPLICATION

For the ABE learner, introductory algebra instruction may be accomplished through an informal exploration of physical models, graphs or other mathematical data. Elementary equations can be generated from number facts with which the learner is familiar and then lead to algebraic representation of those facts. For example, the teacher may say: "I'm thinking of a number which added to two will give me six." Then, writing in symbols:

$$2 + [_\,] = 6$$

$$2 + ? = 6$$

can lead to:

$$2 + N = 6$$

Pattern blocks and Cuisenaire rods are especially helpful in concretely representing algebraic equations. Emphasis should be placed on generalization of patterns and regularly occurring information and phenomena found in everyday life.

In the ESL classroom, algebra should be viewed as another facet of the language of math, and importance should be placed on creating instructional situations where learners may orally analyze, evaluate, and communicate questions and answers relating to the study of algebra basics.

In the workplace, the curriculum should include the teaching of algebra when job tasks of the learner involve these skills. These could include using specific formulas on the job or the reading of dials and meters which use positive and negative numbers. Special effort should be made to analyze the basic math skill requirements of the job and to find actual work-specific materials and problems related to the use of algebra for problem solving. A student in Donna Curry's Workplace Math class wrote in her journal about the link between algebra and quality control:

"S.D. is Standard Deviation. It tells me the boundaries (range) of quality...how each product differs from each other...the margin of error...the margin of perfection. By plotting your findings on an histogram, you can get a clear picture of how [one] product deviates from [an]other..."

For adult diploma students, the ability to translate and represent word problems in algebraic terms is an integral part of many mathematics certification tests. Examinations expects the learner to choose and apply appropriate formulas to solve such test items as problems on perimeter, area, and volume. In addition, the examinee is also expected to understand algebraic notation in describing particular or generalized solutions to problems, such as in the GED "set-up" items. But because this learner is often grounded in concrete, day-to-day realities, algebra may seem overly abstract and difficult to grasp, and may be seen as having little practical use beyond the test. Curriculum efforts should promote a vision where algebra is seen as a bridge between the specifics of arithmetic and more broadly generalized mathematical situations.

SUMMARY

In the adult basic education classroom, curriculum design must include approaches to teaching algebra which allow the learner to:

- ✔ represent arithmetic patterns and real-world situations using tables, graphs, verbal rules, equations, and explore the interrelationships of these presentations;
- ✔ understand the concepts of and recognize the use of variables, expressions and equations;
- ✔ develop confidence in solving equations in one or two variables using concrete, informal, and formal methods;
- ✔ apply algebraic methods to solve and represent a variety of test-related, work-specific or real-world mathematical problems.

STANDARD 9:
STATISTICS AND PROBABILITY

RATIONALE

Adults are bombarded daily with results from statistical studies that can and do impact their lives. The adult learner is frequently aware that such numbers are continually used to define our existence, and generally displays a healthy interest in learning or re-learning the processes used for reaching such conclusions.

In March, Shelley brought in two graphs based on physical fitness. It examined the Healthy People 2000 Campaign which plans to reduce the number of Americans who are defined as medically "sedentary," no leisure time exercise. One related the national, state, and Year 2000 goal pecentages. The other related percentages for the state's most sedentary: those 65 and older, blacks, those with low income, and Hispanic women.

The class exploded! Evelyn in particular was angered by this graph. She said, "They never asked me. Who did they ask?" Another student pronounced, "They are always putting black people down." Ethel said, "They don't know what black people do in their houses. If they know what's going on, why don't they help us?"

They know that decisions made on the basis of statistics may effect them daily, especially in discussions surrounding average scores required for tests, percentage of attendance at work or school, the price of transportation, the cost of child or health care, or even the number of immigrants allowed to become legal residents. It is imperative the adult basic education mathematics student understand not only how such statistical representations are often used, but also how such information may be misused.

APPLICATION

Investigation into statistics and probability should actively engage learners in exploring events and making predictions about situations relevant to their daily lives. In the ABE classroom, simple data related to peers or from other localized sources may be gathered and analyzed. Or students could explore the meaning of the probabilities inherent in lottery games or in the risks of disease as reported in medical journals.

For ESL learners, a study of statistics might also include a study of how the census works and how it is

linked to the political process. And in the workplace, recent trends in the area of quality management, such as statistical process control, are beginning to impact workers at all levels. Workplace instructor Donna Curry describes one of her on-site classes:

"Prep Math [was] the final class in a series of four courses designed to bring employees from absolutely no sense of math up to a level where they would have enough math skills to successfully pass the math portion of the Certified Quality Technician (CQT) test. [The course] covered the following topics: Notation, Metrics, Geometry, Probability, Histograms, and measures central tendencies and...dispersion."

"The company...encouraged employees to become certified...because it felt the national recognition would lend credibility to the individual as well as to the company."

ASE/GED students need to develop a wide range of graphing, data collection and predication competencies as well. These skills are essential for success on the science and social studies portion of the test, and are also invaluable in the pursuit of further educational goals beyond the adult diploma.

SUMMARY

In the adult basic education classroom, curriculum design must include approaches to teaching statistics and probability which allow the learner to:

- ✔ Systematically collect, organize and describe data;
- ✔ Construct, read and interpret tables, charts and graphs;
- ✔ Make inferences and convincing arguments that are based on data analysis;
- ✔ Evaluate arguments that are based on data analysis;
- ✔ Develop an appreciation for statistical methods as a powerful means for decision making.

STANDARD 10:
GEOMETRY AND SPATIAL SENSE

RATIONALE

Adult learners who attend basic mathematics classes at any level share a wealth of pragmatic experience surrounding geometric and spatial concepts. They've probably built a bookcase, laid out a garden, applied wallpaper or tiled a floor, all the while discovering informally the rules which formally govern the study of geometry itself. For many adult students, geometry is one math topic that immediately makes sense to them and gives them confidence in their ability to learn. Therefore, it is important that we cultivate this inherent spatial sense at all levels of adult basic mathematics education.

APPLICATION

In the adult basic education mathematics curriculum importance should be placed on providing learning activities which reflect the concepts and properties of geometry itself: the use of three-dimensional hands-on models; access to manipulable computer software programs which examine the nature of plane and solid figures; and problem solving experiences that allow for physical exploration of the effects of change in angle and measurement.

At the ABE level, learners need to investigate, experiment and explore with everyday objects and other physical material. ABE teacher Linda Huntington centered her whole teacher research project on a geometric theme. Her students combined art and math by designing and making individual quilt squares:

"We were constantly engaged in problem solving: deciding [on] patterns...[determining] which piece to attach to which piece and how. Each student had to measure carefully. They really had to think about and conceptualize the meaning of one half. All the students [many were ESL] learned some new vocabulary... symmetry, diagonal, horizontal, and vertical."

For GED learners, there is a specific and more formal need to understand the concepts of geometry. Instruction must enable them to comprehend and solve multi-step test problems requiring geometric knowledge.

Often workplace education students must also be made aware of geometric principles, since many of their on-the-job tasks (especially in the manufacturing sector), require daily application of common geometric rules.

SUMMARY

In the adult basic education classroom, curriculum design must include approaches to teaching geometry and spatial sense which allow the learner to:

- ✔ Use geometry as a means of describing the physical world relative to all the arts and sciences;
- ✔ Understand and apply geometric properties and relationships to concrete situations;
- ✔ Visualize and represent geometric figures with special attention to developing spatial sense;
- ✔ Identify, describe, compare, and classify geometric figures;
- ✔ Relate geometric ideas to number and measurement ideas, including the concepts of perimeter, area, volume, angle measure, capacity, weight, and mass;
- ✔ Explore transformations of geometric figures;
- ✔ Represent and solve problems using geometric models;
- ✔ Apply the use of appropriate technologies to the study of geometry and spatial sense.

STANDARD 11: **MEASUREMENT**

RATIONALE

Measurement is an essential life skill. Adults use measurement in many familiar con texts: on-the-job, for home improvement projects, in the daily task of food preparation. Despite the fact that competency in measurement is vital, some adult basic education learners have difficulty selecting and determining appropriate units of measure as well as using the appropriate tools of measurement.

APPLICATION

Estimation as well as precision and accuracy are essential components of the skill of measurement. The teaching of estimation helps students gain a firm sense of relative size or quantity. Hands-on use of real measuring tools promotes spatial awareness and understanding of geometric concepts. Students should be exposed to a variety of measurement concepts so that these skills may be applied to other academic disciplines.

In the ABE classroom, teachers should use concrete activities (with non-standard and standard units) to help ABE learners develop an understanding of the many measurable attributes of physical objects (length, time, temperature, capacity, weight, mass, area, volume, and angle). This is a natural way of building a vocabulary for measurement, and for comprehension of what it means to measure.

For ESL learners, teaching measurement is very important as a cross-cultural component of mathematics and second-language learning, since many of these learners have used the metric measurement system more than the customary U.S. system.

Extending measuring skills to concept areas such as perimeter, area, and volume, centigrade and fahrenheit, etc., enables all students (and in particular adult diploma learners) to develop better and more efficient procedures using formulas for solving everyday problems. Concepts of rates and the uses of proportion can be taught to solve direct and indirect measuring problems. Problem situations should be presented for

students to make and use measurements in everyday situations.

In workplace education, instruction should focus on the measurement systems necessary to workplace needs. Teacher Judi Sulzbach describes her on-site class at a company where the metric system of measurement was becoming important:

"I wanted to see what my class knew about the word "metrics', so we brainstormed a few ideas. I asked them to think of any word or phrase that came to mind when they thought of metrics. Some of their ideas were 'groceries,' 'millimeter,' 'grams,' soda,' 'measurement,' 'charts,' and 'measuring tools.' These showed...that they were familiar with a lot more ideas on metric [measurement] than they realized."

"I gave each of them a ruler with cm-mm on one side and inches on the other We set out measuring everything we could: chalk, erasers, pencils, books, paper clips, blackboard, wheel diameter, thickness and hole using all three of the measurements. We then looked at 2 liter soda bottles, soda cans with labeled "ml," and [quart and gallon] milk cartons. [This allowed them to] get an idea of how metric compares with our English measures....Later, we would do the same] with some of the measuring tools that they use on-the-job."

Measurement concepts will become more meaningful if their use in diverse topics (population statistics, magnitudes of galaxies, cells, or countries) is developed.

SUMMARY

In the adult basic education classroom, curriculum design must include concrete and experiential approaches to teaching measurement which allow the learner to:

- ✔ understand the process and concepts of measurement;
- ✔ make and use exact and estimated measurements to describe and compare phenomena;
- ✔ select appropriate units and tools to measure to the degree of accuracy required in a particular situation;
- ✔ understand the structure and use of different systems of measurement.

STANDARD 12:
EVALUATION AND ASSESSMENT

INTRODUCTION

Because passing a diploma-related test is a goal for many adult learners, the existence of the test greatly influences curricular considerations. Several agencies that fund adult basic education programs require standardized tests such as the TABE (Test of Adult Basic Education) to assess learner progress and program success. However, the practice of only using standardized tests to evaluate student growth is frequently disdained by adult education practitioners. There are many reasons why adult educators look to other methods in addition to standardized tests to inform their instructional practice and to diagnose learners' initial skills and progress. Some are philosophical while others are simply pragmatic.

The principles of adult learning come heavily into play as we attempt to assess growth in mathematics mastery with adults. Adults as students are very often independent thinkers whose motivation for learning is internal. While certain external societal pressures do exist which may cause adults to wish to continue their education, most individuals who do attend basic education math classes do so in order to meet self-defined personal or work-related goals. The diploma test outcome aside, these goals bear no relation to passing an exam for a grade or passing from one grade to another. The larger, more overriding wish of the adult basic education population is to "make up" for learning experiences missing, lost or abandoned at earlier stages in the learner's life. The maturity of these adults, plus the pool of life experience that each individual brings to the classroom, demands that a partnership for evaluation be forged between the learner and learning provider.

DESIGN AND METHODOLOGY

In order for students to own the success of mastery, they must also be partners in defining the benchmarks of progress; educator-student dialogues are crucial. For lasting transfer of newly formed skills, the learning experiences must be grounded in immediate real life settings which reflect the complexity of today's world.

One-dimensional computation tests do not begin to measure the growth of an adult student as she/he progresses through those magical math "A-HA!'s" that comes from having shared a problem in a group or with a friend. Single step, numbers-only computation no longer addresses the complex math challenges adults face every day. Evaluation of adult learning in mathematics must stem from a learner-centered curriculum which reflects multiple strategies for showing evidence of mastery. Essays, discussions, drawings and model design — all including and involving the use of appropriate problem solving techniques — should accompany traditional computation exercises in order to help frame the educator-learner dialogue regarding progress and goal attainment.

Varied adult mathematics populations require that assessment be in line with the unique characteristics of each group of learners. For the adult basic education learner, (and indeed for learners in general), assessment should be congruent not only with the developmental level of the student but also with respect to the maturity and life experience of the individual.

For the ESL learner, the use of English language math assessment instruments poses a number of barriers. First is the language itself, especially in the case of word problems. Second, the mathematical notation used in the assessment may be foreign to the student. In addition, second language learners may not be familiar with the multiple choice format or be able to perform successfully given the language barrier combined with testing time limits. Methods and tasks for assessing ASE student learning must be geared to the short and long-term goals of the student. While it is essential that the student become comfortable with the content and skills necessary for passing an exam, instruction and assessment should take a broad view in relation to real-life problem solving skills which will serve the student beyond the objective of passing a test.

In the workplace, instructional planning and assessment must represent the needs and goals of these

partners in learning: the learner, the company, and the education provider. They should work in concert to see that there is a clear definition of the goals related to work-related math instruction, and to define how these objectives will be manifested back on the job.

ALIGNMENT

Today's work and life climates demand multiple and integrated sets of mathematical skills. Adults must be able to call on sophisticated problem solving abilities in order to address daily math related problems at home and at work. They must be able to use the technological tools society now regards as commonplace, and to understand that calculators and computers serve as implements for the extension and enhancement, rather than the replacement, of a strong base of problem solving and reasoning skills.

Given this "new world" where mathematics takes place, the methodology for assessing acquisition of these integrated skills must also reflect change. ABE math students may need to learn basic math facts, but they must also be able to translate those abilities to the use of a calculator—a new set of math skills in itself. Performance tasks must be designed to accommodate this demand for an integrated set of skills. English as a Second Language students include language acquisition as well as math skills development in defining their educational progress. Our three-dimensional world encompasses a language which cannot only be reflected on the plane of the workbook page; learners must be able to access the language of math through experience with three dimensional objects. We must design assessment tasks which can incorporate and evaluate the learning expressed via the use of manipulatives. The GED is designed in part to measure computational skills, but does not allow the use of calculators; there appears to be a need on a wider level for the test makers to reassess this decision. Given today's complex world, the higher-order thinking skills should be more heavily stressed and manifested in multi-step problems using calculators. In these instances, a command of the basics

would be inherent in the question. The student would determine *not how* to add or subtract numbers, but *when* to employ such functions. With the aid of the calculator, more time could (and should) be spent addressing the critical thinking aspects of the test item. In workplace math education, assessment tools and tasks must vary from site to site. Measures of success at one workplace may be totally inappropriate to another. There should be a concerted attempt to define mastery in relation to the needs of the specific company and worker; assessment tasks should involve all math topics and tools which would help learners deal with any or all new and emerging technologies.

MULTIPLE SOURCES OF INFORMATION.

Most students come to the adult basic education math classroom with a varied combination of math strengths and weaknesses, along with a collection of math compensation techniques developed over years of functioning as an adult in the real world. It is essential that the math educator develop ways to assess the strengths, shore up the weaknesses, and exploit the often creative compensation techniques. These three factors should form the basis for planning evaluation methods and tasks which encompass a broad range of informative sources relative to measurement of mathematics learning.

For the ABE learner who may be a low level reader or writer but who has good verbal skills, tasks to show mastery might involve one-on-one or small group discussion settings. Conversely, the ESL math learner may be better able to show on paper what he or she is unable to share verbally. Problems addressed in the GED classroom, and indeed on the test, itself should approach similar types of problems in a variety of different printed formats: diagrams and charts versus narrative problems, problems involving narrative answers, or responses requiring drawings versus a straight calculation, multiple choice answer. In the workplace, simulation learning is extremely appropriate; it gives the learner a safe environment in which to practice and finally show mas-

tery of the mathematical task related to the job. In any case, measurement of learning in the adult basic education classroom must reflect a variety of instances and formats where mastery may be shown.

SUMMARY

In the adult basic education classroom, methods and tasks for assessing students' learning should be aligned with the learner's and the curriculum's:

- ✔ goals, objectives, and mathematical content; relative emphasis given to various topics and processes and their relationships; and
- ✔ instructional approaches and activities, including the use of calculators, computers and manipulatives;

The use of assessment data for purposes other than those intended is inappropriate.

Decisions concerning the students' learning should be made on the basis of a convergence of information obtained from a variety of sources. These sources should encompass tasks that

- — demand different kinds of mathematical thinking;
- — present the same mathematical concept or procedure in different contexts, formats, and problems situations;
- — accept and accommodate the wide range of problem solving strategies represented by a diverse learner population;
- — allow for the appropriate use of three dimensional materials to show learning;
- — involve the use of calculators or computers as tools for demonstrating thinking and problem solving skills;
- — include the use of visual, pictorial or graphic representations or models for showing learning;
- — promote the use of verbal skills as a means for developing and displaying problem solving strategies.

BIBLIOGRAPHY

California Department of Education, *Mathematics Frameworks for California Public Schools*, Sacramento, California, 1992.

Cuevas, Gilbert, "Increasing the Achievement and Participation of Language Minority Students in Mathematics Education, Teaching and Learning Mathematics in the 1990's," *1990 Yearbook*, National Council of Teachers of Mathematics, 1991.

Gall, Iddo, *Issues and Challenges in Adult Numeracy*, National Center on Adult Literacy, University of Pennsylvania, (Unpublished Draft Technical Report) January 1993.

Massachusetts Dept. of Education, Bureau of Adult Education, *Handbook for ABE Practitioners*, 1992.

_____, "Mathematics", *Teachers, Tools and Techniques: A Handbook for Adult Basic Education and GED Instructors*, Adult Learning Resource Center, Des Plaines, Illinois, 1991.

National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics*, Reston, Virginia, 1989.

National Council of Teachers of Mathematics, *Professional Standards for Teaching Mathematics*, 1991.

National Council of Supervisors of Mathematics, "Essential Mathematics for the Twenty-first Century: The Position of the National Council of Supervisors of Mathematics," *Arithmetic Teacher*, Vol. 37, No. 13, pp. 44-46, September 1990.

Swartz, Richard et al., *The Official Teacher's Guide to the Tests of General Educational Development*, GED Testing Service, American Council on Education, 1988.